1. This problem is recognizable but undecidable.

**Proof it’s recognizable**: we can write a program that, when given input , simulates on input and reads the state of on every step, seeing what the value of is. If in any of the steps, then return true.

**Proof it’s undecidable**: Let’s make this problem stronger. Let’s define . Assume this problem is decidable, then there exists a Turing machine that, when given input , will halt if and only if will reach state on input . We will reduce from the halting problem, which we recall to be . Let’s make a new Turing machine that converts input into , where is the halting state of , which we define to be . simulates on input . We see that is a decider for the halting problem, but that’s undecidable. This contradiction shows our assumption of the existence of is wrong, which means this problem is undecidable.

1. This problem is recognizable but undecidable.

**Proof it’s recognizable**: we can write a program that, when given input , simulates on input . If it halts, then return true.

**Proof it’s undecidable**: Assume this problem is decidable, then there exists a Turing machine that, when given , will halt if and only if halts when run on .

Wait actually this might be decidable. For each of the control flow statements, it is easy to check whether they will loop forever by checking the loop invariants and see if there is progress towards termination (Dr. Gries yay!).

1. unrecognizable (reduce from co-halting problem)

This problem is unrecognizable. Assume this problem is recognizable, then there exists a Turing Machine that, when given , will accept if both run on and run on don’t halt. We will reduce from the co-halting problem, which we recall to be . Let’s make a new Turing machine that converts input into and simulates on . Observe that is a decider for the co-halting problem, which we know to be unrecognizable. This contradiction shows our assumption of the existence of is wrong, which means this problem is unrecognizable.

1. decidable (just brute force it)